| Table 4 (cont.) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $h$ | $k$ |  | $\left\|F_{o}\right\|$ | $F_{c}$ | $h$ | $k$ |  | $\left\|F_{o}\right\|$ | $F_{c}$ | $h k$ |  | $\left\|F_{o}\right\|$ | $F_{c}$ | $h k$ | $l$ | $\left\|\boldsymbol{F}_{0}\right\|$ | $\boldsymbol{F}_{\boldsymbol{c}}$ |
|  |  | 10 | 55 | 36 |  |  | 2 | 55 | 52 |  | 10 | < 54 | - 5 |  | 3 | 81 | 83 |
|  |  | 11 | 75 | 96 |  |  | 3 | <34 | 4 |  | 11 | 56 | -53 |  | 4 | < 54 | -42 |
|  |  | 12 | 57 | -49 |  |  | 4 | 83 | 94 |  | 12 | < 51 | -10 |  | 5 | 57 | -57 |
|  |  | 13 | 54 | -66 |  |  | 5 | $<38$ | 18 |  | 13 | 51 | $-16$ |  | 6 | < 54 | - 8 |
|  | 14 |  |  | 55 | 21 |  |  | 6 | 106 | $-138$ |  |  |  |  |  | 7 | 57 | -24 |
|  |  |  |  |  |  |  | 7 | $<50$ | -42 | 010 | 0 | 110 | -121 |  | 8 | < 54 | 27 |
| 0 | 7 | 1 | 79 |  | -54 |  |  | 8 | 55 | 83 |  | 1 | < 50 | 12 |  | 9 | 55 | 53 |
|  |  | 2 | 82 | -92 |  |  | 9 | $<54$ | -18 |  | 2 | 58 | 46 |  |  |  |  |
|  |  | 3 | 141 | 160 |  |  | 10 | 57 | 25 |  | 3 | < 51 | -11 | 012 | 0 | 57 | -73 |
|  |  | 4 | 119 | 103 |  |  | 11 | < 54 | 32 |  | 4 | 55 | 30 |  | 1 | 73 | 81 |
|  |  | 5 | 68 | -67 |  |  | 12 | 74 | -74 |  | 5 | < 53 | -43 |  | 2 | 57 | 16 |
|  |  | 6 | 36 | - 6 |  |  | 13 | $<51$ | -32 |  | 6 | 75 | $-75$ |  | 3 | < 57 | -16 |
|  |  | 7 | 41 | -16 |  |  |  |  |  |  | 7 | < 54 | 25 |  | 4 | 57 | 21 |
|  |  | 8 | 66 | -47 | 0 | 9 | 1 | 63 | -59 |  | 8 | 57 | 22 |  | 5 | < 57 | -34 |
|  |  | 9 | 77 | 67 |  |  | 2 | $<38$ | -26 |  | 9 | < 54 | -28 |  | 6 | 56 | -39 |
|  |  | 10 | 75 | 73 |  |  | 3 | 142 | 142 |  | 10 | 56 | 51 |  | 7 | < 55 | 52 |
|  |  | 11 | 57 | $-60$ |  |  | 4 | $<48$ | 21 |  | 11 | < 51 | $-7$ |  |  |  |  |
|  |  | 12 | 54 | - 8 |  |  | 5 | 72 | -68 |  | 12 | 73 | -63 | 013 | 1 | 57 | - 13 |
|  |  | 13 | 55 | $-20$ |  |  | 6 | $<51$ | -34 |  | 13 | <46 | 4 |  | 2 | < 56 | 52 |
|  | 14 |  | 50 | -36 |  |  | 7 | 55 | 5 |  |  |  |  |  | 3 | 56 | 28 |
|  |  |  |  |  |  | 8 | < 54 | -21 | 011 | 1 | 78 | -55 |  | 4 | < 56 | -42 |
| 0 | 8 | 0 |  | 103 | $-105$ |  |  | 9 | 81 | 73 |  | 2 | < 53 | 29 |  | 5 | 55 | -16 |
|  |  | 1 | 74 | -69 |  |  |  |  |  |  |  |  |  |  |  |  |  |

Reynolds who undertook most of the computing work, including the tedious task of finding the best positions for the light atoms.

## References

Alderman, P. R. H., Owston, P. G. \& Rowe, J. M. (1960). Acta Cryst. 13, 149.

Bleidelis, Ya. Ya. (1957). Kristallografiya 2, 278.
Bleidelis, Ya. Ya. \& Bokir, G. B. (1957). Kristallografiya 2, 281.

Cavalca, L., Nardelli, M. \& Fava, G. (1959). Proc. Chem. Soc. 159.
Chatt, J. \& Hart, F. A. (1952). Nature, Lond. 169, 673.
Chatt, J. \& Duncanson, L. A. (1956). Nature, Lond. 178, 997.

Chatt, J., Duncanson, L. A., Hart, F. A. \& Owston, P. G. (1958). Nature, Lond. 181, 43.

Lindquist, I. (1957). Acta Cryst. 10, 29.
Owston, P. G., Partridge, J. M. \& Rowe, J. M. (1960). Acta Cryst. 13, 246.
Zhdanov, G. S. \& Zvonkova, Z. V. (1953). Uspelchi Khimi. 22 (1) 3.

# The Structure of the Intermetallic Phase $\theta(\mathrm{Cr}-\mathrm{Al})$ 

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(Received 31 July 1959)
The monoclinic $\theta$-phase in the chromium-aluminium system has been shown to be isomorphous with $\alpha^{\prime}(\mathrm{V}-\mathrm{Al})$. The structure has been refined and the interatomic distances are discussed and compared with those in $\alpha^{\prime}(\mathrm{V}-\mathrm{Al})$ and with those in two ternary alloy phases containing chromium and aluminium.

## 1. Introduction

The chromium-aluminium system has been investigated by many workers and the composition of the $\theta$-phase, which is in equilibrium with the primary solid solution, has been shown to be represented by the formula $\mathrm{CrAl}_{7}$ (Raynor \& Little, 1945). The phase forms monoclinic crystals and the unit cell given by Hofmann \& Wiehr (1941) is:

$$
a=20 \cdot 43, b=7 \cdot 62, c=25 \cdot 31 \AA ; \beta=155^{\circ} 10^{\prime} .
$$

This unit cell corresponds to the morphology of the crystals.

The similarity of rotation photographs from $\theta(\mathrm{Cr}-\mathrm{Al})$ and $\alpha^{\prime}(\mathrm{V}-\mathrm{Al})$ has been reported by Brown (1957). The present analysis of $\theta(\mathrm{Cr}-\mathrm{Al})$ was undertaken to investigate the relationship between the two phases and in particular the nature of the chromium-aluminium
bonds, since interesting orientational properties have been observed in the abnormally short transition metal to aluminium bonds which occur in certain phases (Brown, 1957).

## 2. Material

The crystals used in this analysis were prepared from an aluminium-chromium alloy containing about $1 \%$ weight chromium. The alloy was cooled from $700^{\circ} \mathrm{C}$. to $500^{\circ} \mathrm{C}$. at a rate of $15^{\circ}$ per hour, and then quenched rapidly. The crystals were extracted electrolytically and found to be in the form of plates and prisms. Many of the plates appeared to be twinned. A photograph of similar crystals is given by Hofmann \& Wiehr. No difficulty was found in selecting single crystals for analysis.

## 3. Unit cell

Laue, oscillation and Weissenberg photographs confirmed that the unit cell is monoclinic and provided approximate values of the lattice parameters. The alternative unit cell given by Hofmann \& Wiehr was used in this analysis since it has only one parameter greater than $11 \AA$ and also corresponds to that of $\alpha^{\prime}(\mathrm{V}-\mathrm{Al})$ used by Brown (1959). It is related to the morphological unit cell by the matrix:

$$
\begin{aligned}
& 001 \\
& 010 \\
& \overline{1} 0 \overline{1}
\end{aligned}
$$

The transformation of the Hofmann \& Wiehr cell gives the new unit cell:

$$
a=25 \cdot 31, b=7 \cdot 62, c=10.92 \AA ; \beta=128^{\circ} 12^{\prime}
$$

Accurate lattice parameters were determined by the $\theta$-method described by Weisz, Cochran \& Cole (1948), but using a Geiger Counter and linear ratemeter in place of the photographic film used by them. The values obtained were:

$$
\begin{gathered}
a=25 \cdot 196 \pm 0 \cdot 017, b=7 \cdot 574 \pm 0 \cdot 003 \\
c=10 \cdot 949 \pm 0 \cdot 009 \AA ; \beta=128^{\circ} 43^{\prime} \pm 3^{\prime}
\end{gathered}
$$

The volume of the unit cell is therefore $1630 \pm 2 \AA^{3}$. The density of the crystals, determined by the displacement method, was $3 \cdot 18 \pm 0.05$ g.cm. ${ }^{-3}$ so that the mass content of the unit cell is $3060 \pm 50$ A.M.U.

A sample of about 50 mg . of the crystals was chemically analysed by Messrs. Johnson, Matthey and Co. Ltd., giving $22 \cdot 6 \%$ weight chromium and $77 \cdot 6 \%$ weight aluminium. This corresponds to a ratio of one chromium atom to 6.6 aluminium atoms. The mass of the $\mathrm{CrAl}_{6.6}$ unit is 230 A.M.U., so that there are 14 chromium atoms and approximately 90 aluminium atoms in the unit cell.

## 4. The space group

Laue and Weissenberg photographs indicated that the
unit cell is $C$-face centred and that the Laue symmetry group is $2 / m$. The possible space groups are therefore $C 2, C m$ and $C 2 / m$. These can be distinguished by the presence or absence of a centre of symmetry in the [010] and [001] projections, since $C 2$ has a centre of symmetry in the [010] projection only, $C m$ has no centre of symmetry in either projection, and $C 2 / m$ has a centre of symmetry in both projections. A Wilson statistical test was therefore applied to intensities from Weissenberg photographs for the [010] and [001] zones, and the results of this are shown in Fig. 1. In both cases the points lie above the curve for a centro-symmetric distribution and the space group $C 2 / m$ was therefore chosen.


Fig. 1. Tests for centro-symmetry of the [001] and [010] zones.

## 5. Collection of intensities

The crystal used for the collection of intensity data was approximately cubic with edges about 0.1 mm . in length and was cut from one of the prismatic single crystals. For this size of crystal the correction of intensities for absorption of Mo $K \alpha$ radiation can be assumed to be a function of $\sin \theta / \lambda$ only, within the accuracy of the measurements.

Reflections of the form ( $h 0 l$ ) were collected on a zero-layer normal-beam Weissenberg photograph using filtered Mo $K \alpha$ radiation and a multiple-film technique. An intensity scale was made using a reflection in this zone from the same crystal and intensities out to $\sin \theta / \lambda=1.0$ were measured by visual comparison with this scale. The intensities were corrected for Lorentz and polarization factors and allowance was made for absorption and resolution of the $\alpha_{1} \alpha_{2}$ doublet by scaling $F_{o}$ to $F_{c}$ in regions of $\sin \theta / \lambda$. A rough correction for extinction was found to be necessary for the stronger reflections.

Reflections of the form $h k 0$ were collected in a similar way. Due to the large amount of overlap in the [010] projection it was found necessary to collect reflections of the form $h k l$ and $h k 2$ to complete the refinement of the structure. These were collected on


Fig. 2. Final $F_{o}$ and $F_{o}-F_{c}$ [010] Fourier projections. (a) $F_{o}$. The contours are at equal arbitrary intervals. The $F^{\prime}(000)$ term is not included. (b) $F_{o}-F_{c}$. The contour intervals are one-fifth those of the $F_{o}$; negative contours are shown as broken lines. Squares represent atomic sites.
two equi-inclination Weissenberg photographs and their intensities measured as before.

## 6. Isomorphism with $\alpha^{\prime}(\mathrm{V}-\mathrm{Al})$

The similarity of rotation photographs from $\theta(\mathrm{Cr}-\mathrm{Al})$ and $\alpha^{\prime}(\mathrm{V}-\mathrm{Al})$ has been reported by Brown (1957). The similarity of the structures was also indicated by a comparison of the photographs obtained in this analysis and those obtained by Brown (1959) in the analysis of $\alpha^{\prime}(\mathrm{V}-\mathrm{Al})$. The cell dimensions and contents were also found to be very similar and the space group is the same. It was therefore decided to take the structure of $\alpha^{\prime}(\mathrm{V}-\mathrm{Al})$, which was then reasonably well refined, as a trial structure for $\theta(\mathrm{Cr}-\mathrm{Al})$, replacing the vanadium atoms by chromium atoms. The values of $F_{o}$ for reflections out to $\sin \theta / \lambda=0.5$ in the [010] zone and to $\sin \theta / \lambda=0.8$ in the [001] zone were scaled in regions of $\sin \theta / \lambda$ to the values of $F_{c}$ from this trial structure and $R$-factors of 0.15 and 0.18 respectively were obtained, confirming the similarity of the structures.

The positions of the chromium atoms are 2 in $2(a)$, 4 in $4(i)$ and 8 in $8(j)$ and the positions of the aluminium atoms are 2 in $2(d), 8$ sets of 4 in $4(i)$ and 7 sets of 8 in $8(j)$, making a total of 14 chromium and 90 aluminium atoms in the unit cell.

## 7. Refinement of the [010] projection

The refinement of the structure was carried out by
means of $F_{o}-F_{c}$ syntheses, using the atomic scattering factors given by Forsyth \& Wells (1959). After a few cycles using reflections out to $\sin \theta / \lambda=0 \cdot 5$, the further reflections out to $\sin \theta / \lambda=0.8$ were included. An additional isotropic temperature factor of $B=0.40$ was applied to all the aluminium atoms in addition to the factor of 0.24 introduced by the scaling, which applies to all atoms. The final $R$ factor was 0.060 for 350 measured reflections.

In this projection all atoms are well resolved with the exception of $\mathrm{Cr}_{2}$ and $\mathrm{Al}_{7}$ and since the contribution from the single $\mathrm{Al}_{7}$ atom is much less than that from the double $\mathrm{Cr}_{2}$ atom it is only for this atom that the $x$ and $z$ co-ordinates cannot be obtained with so much accuracy. The final $F_{o}$ and $F_{o}-F_{c}$ syntheses are shown in Fig. 2.

## 8. Refinement of the [001] projection

While the [010] projection was being refined, the [001] projection was refined independently using reflections from the zero layer out to $\sin \theta / \lambda=0.8$ and the $R$-factor was reduced to about $0 \cdot 10$. A large amount of overlap occurs in this projection and in order to overcome this intensities of reflections of the form ( $h k 1$ ) and ( $h k 2$ ) were collected.

For the space group $C 2 / m$, the structure factor for a reflection $h k l$ is given by:

# Table 1. Atomic co-ordinates for $\theta(\mathrm{Cr}-\mathrm{Al})$ and the corresponding values of $\cos 2 \pi l z$ for the first and second layers 

| Final co-ordinates |  |  | $\cos 2 \pi l z$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $x$ | $y$ | $z$ | $l=1$ | $l=2$ |
|  |  |  | 1.000 | 1.000 |
| $0.2505 \pm 0.0002$ |  | $0 \cdot 2630 \pm 0.0004$ | -0.082 | $-0.987$ |
| $0 \cdot 0854 \pm 0.0001$ | $0 \cdot 3240 \pm 0.0003$ | $0.7525 \pm 0.0002$ | 0.054 | -0.994 |
|  |  |  | -1.000 | 1.000 |
| $0 \cdot 6152 \pm 0.0003$ |  | $0.0005 \pm 0.0006$ | 1.000 | 1.000 |
| $0 \cdot 2932 \pm 0.0003$ |  | $0.0745 \pm 0.0006$ | 0.893 | $0 \cdot 593$ |
| $0 \cdot 3750 \pm 0.0003$ |  | $0 \cdot 3840 \pm 0.0006$ | -0.745 | $0 \cdot 111$ |
| $0.5214 \pm 0.0003$ |  | $0 \cdot 2870 \pm 0.0006$ | -0.230 | -0.894 |
| $0.0810 \pm 0.0006$ |  | $0.7235 \pm 0.0020$ | -0.165 | -0.945 |
| $0 \cdot 1285 \pm 0.0003$ |  | $0 \cdot 1445 \pm 0.0006$ | 0.616 | -0.242 |
| $0.0912 \pm 0.0003$ |  | $0 \cdot 3410 \pm 0.0006$ | -0.541 | $0 \cdot 413$ |
| $0.2245 \pm 0.0003$ |  | $0 \cdot 4575 \pm 0.0006$ | -0.965 | $0 \cdot 861$ |
| $0 \cdot 1820 \pm 0.0002$ | $0 \cdot 1850 \pm 0 \cdot 0004$ | $0 \cdot 0090 \pm 0.0004$ | 0.998 | 0.994 |
| $0.0692 \pm 0.0002$ | $0 \cdot 3205 \pm 0.0004$ | $0 \cdot 1485 \pm 0 \cdot 0004$ | 0.595 | -0.291 |
| $0.9643 \pm 0.0002$ | $0 \cdot 1880 \pm 0 \cdot 0005$ | $0 \cdot 1330 \pm 0 \cdot 0004$ | $0 \cdot 670$ | -0.101 |
| $0 \cdot 3100 \pm 0.0002$ | $0.3155 \pm 0.0005$ | $0 \cdot 2405 \pm 0.0004$ | 0.059 | -0.993 |
| $0.0456 \pm 0.0002$ | $0 \cdot 1955 \pm 0.0005$ | $0 \cdot 4695 \pm 0.0004$ | -0.982 | 0.927 |
| $0 \cdot 1655 \pm 0.0002$ | $0 \cdot 3070 \pm 0 \cdot 0004$ | $0 \cdot 4760 \pm 0 \cdot 0004$ | -0.978 | 0.915 |
| $0.2065 \pm 0.0002$ | $0.3145 \pm 0.0005$ | $0 \cdot 2780 \pm 0.0004$ | $-0.175$ | -0.938 |

$$
\begin{aligned}
F(h k l)= & \sum_{n} f_{n} \cos 2 \pi(h x+l z) \cos 2 \pi k y \\
= & \sum_{n} f_{n}(\cos 2 \pi h x \cos 2 \pi l z-\sin 2 \pi h x \sin 2 \pi l z) \\
& \times \cos 2 \pi k y
\end{aligned}
$$

and for a reflection $\bar{h} k l$ by:

$$
\begin{aligned}
F(\bar{h} k l)= & \sum_{n} f_{n}(\cos 2 \pi h x \cos 2 \pi l z+\sin 2 \pi h x \sin 2 \pi l z) \\
& \times \cos 2 \pi k y .
\end{aligned}
$$

Therefore
$F(h k l)+F(\bar{h} k l)=2 \sum_{n} f_{n} \cos 2 \pi h x \cos 2 \pi k y \cos 2 \pi l z$.
The Fourier synthesis of $F(h k l)+F(\bar{h} k l)$ perpendicular to the $c$ axis will therefore have peaks at the atomic positions weighted by the corresponding value of $f_{n} \cos 2 \pi l z$. The values of $\cos 2 \pi l z$ for the atoms for the first and second layers are given in Table 1. Only the $y$ co-ordinates of the 8 sets of atoms in general positions remained to be determined from the [001] projection. Of these $\mathrm{Cr}_{2}, \mathrm{Al}_{14}$ and $\mathrm{Al}_{17}$ do not contribute appreciably to the first layer synthesis and $\mathrm{Al}_{12}$ and $\mathrm{Al}_{13}$ do not contribute appreciably to the second layer synthesis, so that these two layers were sufficient to complete the refinement.
The final $R$-factors were as follows:
$\begin{array}{ll}\text { zero layer } & 110 \text { measured reflections } R=0.092 \\ \text { first layer } & 215 \text { measured reflections } R=0.059 \\ \text { second layer } & 220 \text { measured reflections } R=0.057\end{array}$
The final $F_{o}$ and $F_{o}-F_{c}$ syntheses for the first and second layers are shown in Figs. 3 and 4.

The final $R$-factor for all measured reflections with $\sin \theta \mid \lambda$ less than 1.0 (ll34 reflections) was 0.068 . The final atomic co-ordinates with their standard deviations are given in Table 1. The standard deviations


Fig. 3. Final $F_{o}$ and $F_{o}-F_{c}(h k 1)$ Fourier projections. Negative contours are shown as broken lines. (a) $F_{o}$. The contours are at equal arbitrary intervals. (b) $F_{o}-F_{c}$. The contour intervals are one-fifth those of the $F_{o}$. Squares represent sites of atoms which contribute appreciably to the ( $h k 1$ ) $F_{o}$ synthesis.


Fig. 4. Final $F_{o}$ and $F_{o}-F_{c}(h k 2)$ Fourier projections. Negative contours are shown as broken lines. (a) $F_{0}$. The contours are at equal arbitrary intervals. (b) $F_{o}-F_{c}$. The contour intervals are one-fifth those of the $F_{0}$. Squares represent sites of atoms which contribute appreciably to the ( $h k 2$ ) $F_{o}$ synthesis.
have been calculated from the final $F_{o}-F_{c}$ syntheses by the method suggested by Lipson \& Cochran (1953). Tabulated values of $F_{o}$ and $F_{c}$ are available.

## 9. The interatomic distances

The structure is similar to that of $\alpha^{\prime}(\mathrm{V}-\mathrm{Al})$ and Fig. 5 shows several co-ordination groups in their appropriate orientation, indicating how these are linked.

A list of interatomic distances is given in Table 2, together with their standard deviations. The mean $\mathrm{Al}-\mathrm{Al}$ distance is $2.84 \AA$, which is about normal for this type of compound. The $\mathrm{Cr}-\mathrm{Al}$ distances can be divided into two groups having mean values of $2.53 \AA$ and $2.74 \AA$. The first of these groups contains the bonds involved in the linkage between the co-ordination polyhedra. These are all the $\mathrm{Cr}-\mathrm{Al}$ distances less than $2.59 \AA$, with the exception of the $\mathrm{Cr}_{1}-\mathrm{Al}_{5}$ distance ( $2 \cdot 53 \AA$ ). In each case except $\mathrm{Al}_{0}$ the aluminium atom has only two chromium neighbours and the $\mathrm{Cr}-\mathrm{Al}-\mathrm{Cr}$ bond angle is about $180^{\circ} . \mathrm{Al}_{0}$ has four
chromium neighbours, with two pairs of $180^{\circ}$ bonds. No bond angles near $180^{\circ}$ occur in the second group. The sum of the twelve co-ordinated radii of chromium and aluminium is $2.71 \AA$, so that the value $2.53 \AA$ for the first group is significantly shorter than this. The corresponding mean $\mathrm{V}-\mathrm{Al}$ distances in the isomorphous $\alpha^{\prime}(\mathrm{V}-\mathrm{Al})$ are $2.56 \AA$ and $2.79 \AA$ respectively. The smaller values obtained for $\theta(\mathrm{Cr}-\mathrm{Al})$ are as would be expected from the replacement of the vanadium atoms by the smaller chromium atoms.


Fig. 5. Structural groups in $\theta(\mathrm{Cr}-\mathrm{Al})$. The numbers of the atoms refer to those given in Table 1. The polyhedra are shown as if solid, with the chromium atoms added at their centres.

The distance between the $\mathrm{Cr}_{2}$ atoms is $2 \cdot 666 \AA$, which is larger than the corresponding $V-V$ distance ( $2 \cdot 636 \AA$ ) in $\alpha^{\prime}(\mathrm{V}-\mathrm{Al})$ and differs from it in being significantly larger than the closest distance of approach in the pure metal ( $2 \cdot 50 \AA$ ). This would seem to emphasise the importance of the opposing short $\mathrm{Cr}-\mathrm{Al}$ bonds.

The values of the $\mathrm{Cr}-\mathrm{Al}$ distances may also be compared with those in $\mathrm{Cr}_{4} \mathrm{Si}_{4} \mathrm{Al}_{13}$ (Robinson, 1953) and $\mathrm{Mg}_{3} \mathrm{Cr}_{2} \mathrm{Al}_{18}$ (Samson, 1958). The occurrence of short $\mathrm{Cr}-\mathrm{Al}$ distances in the former has been discussed by Taylor (1954) in connection with certain other alloy phases. The atomic co-ordinates given for these structures lead to the $\mathrm{Cr}-\mathrm{Al}$ distances and inter-bond angles which are given in Table 3, together with those for $\theta(\mathrm{Cr}-\mathrm{Al})$. The short distances again occur when the aluminium atom has only two chromium neighbours and the inter-bond angle is about $180^{\circ}$. In both these structures the chromium atoms are co-ordinated by twelve other atoms which lie at the vertices of nearly regular icosahedra. In $\mathrm{Cr}_{4} \mathrm{Si}_{4} \mathrm{Al}_{13}$ these form groups of four inter-penetrating icosahedra, each chromium atom having three chromium neighbours, and these groups share corners with other groups. In $\mathrm{Mg}_{3} \mathrm{Cr}_{2} \mathrm{Al}_{18}$ icosahedra share corners only, but in both cases the short distances are associated with the linkage between co-ordination polyhedra. The values of the short

| Atom | Already quoted | Neigh bours | Distance <br> ( $\AA$ ) | Std. devn. <br> ( $\AA$ ) | Atom | Already quoted | Neigh bours | Distance $(\AA)$ | Std.devn. $(\AA)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Cr}_{0}$ |  | $2 \mathrm{Al}_{8}$ | $2 \cdot 570$ | $0 \cdot 007$ | $\mathrm{Al}_{8}$ | $1 \mathrm{Cr}_{0}, 1 \mathrm{Cr}_{1}$ | $1 \mathrm{Al}_{9}$ | $2 \cdot 834$ | 0.010 |
|  |  | $2 \mathrm{Al}_{9}$ | $2 \cdot 922$ | $0 \cdot 007$ |  |  | $1 \mathrm{Al}_{10}$ | $2 \cdot 696$ | 0.010 |
|  |  | $4 \mathrm{Al}_{12}$ | $2 \cdot 836$ | $0 \cdot 005$ |  |  | $2 \mathrm{Al}_{11}$ | 2.910 | 0.009 |
|  |  | $4 \mathrm{Al}_{13}$ | $2 \cdot 567$ | $0 \cdot 005$ |  |  | $2 \mathrm{Al}_{12}$ | $2 \cdot 865$ | $0 \cdot 006$ |
| $\mathrm{Cr}_{1}$ |  | $1 \mathrm{Al}_{4}$ | $2 \cdot 860$ | $0 \cdot 009$ | $\mathrm{Al}_{9}$ | $1 \mathrm{Cr}_{0}, 1 \mathrm{Al}_{8}$ | $2 \mathrm{Al}_{13}$ | $2 \cdot 807$ | $0 \cdot 009$ |
|  |  | $1 \mathrm{Al}_{5}$ | $2 \cdot 534$ | 0.009 |  |  | $2 \mathrm{Al}_{17}$ | $2 \cdot 845$ | $0 \cdot 006$ |
|  |  | $1 \mathrm{Al}_{8}$ | $2 \cdot 483$ | $0 \cdot 009$ |  |  | $1 \mathrm{Al}_{10}$ | $2 \cdot 747$ | $0 \cdot 010$ |
|  |  | $1 \mathrm{Al}_{10}$ | $2 \cdot 592$ | $0 \cdot 009$ |  |  | $2 \mathrm{Al}_{12}$ | 3.031 | $0 \cdot 006$ |
|  |  | $2 \mathrm{Al}_{11}$ | $2 \cdot 588$ | 0.007 |  |  | $2 \mathrm{Al}_{13}$ | $2 \cdot 893$ | $0 \cdot 009$ |
|  |  | $2 \mathrm{Al}_{14}$ | $2 \cdot 911$ | 0.007 |  |  | $2 \mathrm{Al}_{15}$ | $2 \cdot 739$ | $0 \cdot 009$ |
|  |  | $2 \mathrm{Al}_{16}$ | $2 \cdot 693$ | $0 \cdot 007$ |  |  | $2 \mathrm{Al}_{16}$ | $2 \cdot 766$ | 0.006 |
|  |  | $2 \mathrm{Al}_{17}$ | $2 \cdot 675$ | 0.007 | $\mathrm{Al}_{10}$ | $1 \mathrm{Cr}_{1}, 1 \mathrm{Al}_{8}$$1 \mathrm{Al}_{9}$ | $2 \mathrm{Al}_{16}$ | $2 \cdot 803$ | $0 \cdot 009$ |
| $\mathrm{Cr}_{2}$ |  | $1 \mathrm{Cr}_{2}$ | $2 \cdot 666$ | $0 \cdot 005$ |  |  | $2 \mathrm{Al}_{16}$ | $2 \cdot 831$ | $0 \cdot 006$ |
|  |  | $1 \mathrm{Al}_{0}$ | 2.577 | $0 \cdot 003$ |  |  | $2 \mathrm{Al}_{17}$ | $2 \cdot 667$ | $0 \cdot 009$ |
|  |  | $1 \mathrm{Al}_{3}$ | $2 \cdot 678$ 2.740 | 0.007 0.007 |  |  | $2 \mathrm{Al}_{17}$ | $2 \cdot 939$ | 0.006 |
|  |  | ${ }_{1} \mathrm{Al}_{4}$ | $2 \cdot 740$ $2 \cdot 620$ | 0.007 0.007 | $\mathrm{Al}_{11}$ | $\begin{array}{ll} 1 \mathrm{Cr}_{1}, & 1 \mathrm{Cr}_{2} \\ 1 \mathrm{Al}_{3}, & 1 \mathrm{Al}_{4} \\ \mathrm{l} \mathrm{AI}_{4}, & 1 \mathrm{Al}_{7} \\ 1 \mathrm{AI}_{8} & \end{array}$ | $1 \mathrm{Al}_{11}$ | $2 \cdot 802$ | 0.010 |
|  |  | $1 \mathrm{Al}_{6}$ | $2 \cdot 785$ | 0.007 |  |  | $1 \mathrm{Al}_{13}$ | $2 \cdot 978$ | 0.007 |
|  |  | $1 \mathrm{Al}_{7}$ | $2 \cdot 468$ | 0.003 |  |  | $1 \mathrm{Al}_{14}$ | $2 \cdot 761$ | $0 \cdot 007$ |
|  |  | $1 \mathrm{Al}_{11}$ | $2 \cdot 531$ | 0.006 |  |  | $1 \mathrm{Al}_{14}$ | $\stackrel{2}{2} 861$ | $0 \cdot 007$ |
|  |  | $1 \mathrm{Al}_{13}$ | $2 \cdot 481$ | $0 \cdot 006$ |  |  | $1 \mathrm{Al}_{17}$ | $2 \cdot 784$ | 0.007 |
|  |  | $1 \mathrm{Al}_{14}$ | $2 \cdot 796$ | $0 \cdot 006$ | $\mathrm{Al}_{12}$ | $1 \mathrm{Cr}_{0}, 1 \mathrm{Al}_{3}$ | $1 \mathrm{Al}_{12}$ | 2.719 | 0.007 |
|  |  | $1 \mathrm{Al}_{15}$ | $2 \cdot 772$ | 0.006 |  | $1 \mathrm{Al}_{6}, \mathrm{IAI}_{8}$ | $1 \mathrm{Al}_{12}$ | 2.935 | 0.010 |
|  |  | $1 \mathrm{Al}_{15}$ | $2 \cdot 786$ | $0 \cdot 006$ |  | $1 \mathrm{Al}_{9}$ | $1 \mathrm{Al}_{13}$ | $2 \cdot 732$ | $0 \cdot 007$ |
| $\mathrm{Al}_{0}$ | $4 \mathrm{Cr}_{2}$ | $2 \mathrm{Al}_{5}$ | $2 \cdot 559$ | $0 \cdot 007$ |  |  | $1 \mathrm{Al}_{13}$ | $2 \cdot 825$ | $0 \cdot 007$ |
|  |  | $2 \mathrm{Al}_{6}$ | $2 \cdot 701$ | 0.007 |  |  | $1 \mathrm{Al}_{16}$ | $2 \cdot 814$ 2.806 | $0 \cdot 007$ |
|  |  | $4 \mathrm{Al}_{15}$ | $2 \cdot 688$ | 0.005 |  |  | $1 \mathrm{Al}_{17}$ | $2 \cdot 806$ | $0 \cdot 007$ |
| $\mathrm{Al}_{3}$ | $2 \mathrm{Cr}_{2}$ |  | $2 \cdot 891$ | $0 \cdot 010$ | $\mathrm{Al}_{13}$ | $\begin{array}{lll} 1 \mathrm{Cr}_{0}, & 1 \mathrm{Cr}_{2} \\ 1 \mathrm{Al}_{3}, & 1 \mathrm{Al}_{6} \\ 1 \mathrm{Al}_{7}, & 1 \mathrm{Al}_{8} \\ 1 \mathrm{Al}_{9}, & 1 \mathrm{Al}_{11} \\ 1 \mathrm{AI}_{12}, & 1 \mathrm{Al}_{12} \end{array}$ | $1 \mathrm{Al}_{13}$ | $2 \cdot 848$ | 0.010 |
|  |  | $1 \mathrm{Al}_{6}$ | $2 \cdot 875$ | 0.010 |  |  | $1 \mathrm{Al}_{15}$ | $2 \cdot 894$ | $0 \cdot 007$ |
|  |  | $2 \mathrm{Al}_{11}$ | 2.887 | 0.006 |  |  |  |  |  |
|  |  | $2 \mathrm{Al}_{12}$ | $2 \cdot 856$ | 0.009 |  |  |  |  |  |
|  |  | $2 \mathrm{Al}_{13}$ | $2 \cdot 844$ | $0 \cdot 006$ |  |  |  |  |  |
|  |  | $2 \mathrm{Al}_{17}$ | $2 \cdot 792$ | $0 \cdot 009$ | $\mathrm{Al}_{14}$ | $\begin{array}{ll} 1 \mathrm{Cr}_{1}, & 1 \mathrm{Cr}_{2} \\ 1 \mathrm{Al}_{4}, & 1 \mathrm{Al}_{4} \\ 1 \mathrm{Al}_{5}, & 1 \mathrm{Al}_{7} \\ 1 \mathrm{Al}_{11}, & 1 \mathrm{Al}_{11} \end{array}$ | $1 \mathrm{AL}_{14}$ | $2 \cdot 794$ | 0.010 |
| $\mathrm{Al}_{4}$ | l $\mathrm{Cr}_{1}, 2 \mathrm{Cr}_{2}$,$1 \mathrm{Al}_{3}$ | $1 \mathrm{Al}_{5}$ | $2 \cdot 652$ | $0 \cdot 010$ |  |  | $1 \mathrm{Al}_{15}$ | $2 \cdot 990$ | 0.007 |
|  |  | $2 \mathrm{Al}_{11}$ | $2 \cdot 762$ | $0 \cdot 006$ |  |  | ${ }_{1} \mathrm{Al}_{16}$ | 2.914 2.881 | 0.007 0.007 |
|  |  | $2 \mathrm{Al}_{11}$ | $2 \cdot 797$ | 0.009 |  |  | $1 \mathrm{Al}_{1}{ }^{-}$ | $2 \cdot 881$ | $0 \cdot 007$ |
|  |  | $2 \mathrm{Al}_{14}$ | 2.869 | $0 \cdot 006$ | $\mathrm{Al}_{15}$ | $1 \mathrm{Cr}_{2}, 1 \mathrm{Cr}_{2}$ | $1 \mathrm{Al}_{15}$ | $2 \cdot 763$ | 0.007 |
|  |  | $2 \mathrm{Al}_{14}$ | $3 \cdot 073$ | $0 \cdot 009$ |  | $1 \mathrm{Al}_{0}, 1 \mathrm{Al}_{5}$ | $1 \mathrm{Al}_{15}$ | 2.961 3.095 | 0.010 0.007 |
| $\mathrm{Al}_{5}$ | $\begin{array}{ll} 1 \mathrm{Cr}_{1}, & 2 \mathrm{Cr}_{2} \\ 1 \mathrm{Al}_{0}, & 1 \mathrm{Al}_{4} \end{array}$ | $1 \mathrm{Al}_{6}$ | $2 \cdot 843$ | $0 \cdot 010$ |  | $\begin{array}{ll} 1 \mathrm{Al}_{6}, & 1 \mathrm{Al}_{7} \\ 1 \mathrm{Al}_{3}, & 1 \mathrm{Al}_{0} \end{array}$ | $1 \mathrm{AI}_{16}$ | $3 \cdot 095$ | $0 \cdot 007$ |
|  |  | $2 \mathrm{Al}_{14}$ | $2 \cdot 767$ | $0 \cdot 006$ |  | $1 \mathrm{Al}_{7}, \quad 1 \mathrm{Al}_{9}$ |  |  |  |
|  |  | $2 \mathrm{Al}_{15}$ | $2 \cdot 810$ | 0.006 |  | $1 \mathrm{Al}_{13}, 1 \mathrm{Al}_{14}$ |  |  |  |
|  |  | $2 \mathrm{Al}_{16}$ | $2 \cdot 734$ | $0 \cdot 009$ | $\mathrm{Al}_{16}$ | $1 \mathrm{Cr}_{1}, 1 \mathrm{Al}_{5}$ | $1 \mathrm{Al}_{16}$ | $2 \cdot 923$ | $0 \cdot 010$ |
| $\mathrm{Al}_{6}$ | $\begin{array}{lll} 2 \mathrm{Cr}_{2}, & 1 \mathrm{Al}_{0} \\ 1 \mathrm{Al}_{3}, & 1 \mathrm{Al}_{5} \end{array}$ | $2 \mathrm{Al}_{12}$ | $2 \cdot 806$ | $0 \cdot 009$ |  | $1 \mathrm{Al}_{6}, \mathrm{l} \mathrm{Al}_{9}$ | $1 \mathrm{Al}_{17}$ | 2.772 | $0 \cdot 007$ |
|  |  | $2 \mathrm{Al}_{13}$ | $2 \cdot 734$ | 0.006 |  | $1 \mathrm{Al}_{10}, 1 \mathrm{Al}_{10}$ | $1 \mathrm{Al}_{17}$ | $2 \cdot 925$ | $0 \cdot 007$ |
|  |  | $2 \mathrm{Al}_{15}$ | $2 \cdot 857$ | $0 \cdot 006$ |  | $1 \mathrm{Al}_{12}, \mathrm{l} \mathrm{Al} 14$ |  |  |  |
|  |  | $2 \mathrm{Al}_{16}$ | $3 \cdot 201$ | $0 \cdot 009$ |  | $1 \mathrm{Al}_{15}$ |  |  |  |
| $\mathrm{Al}_{7}$ | $2 \mathrm{Cr}_{2}$ | $2 \mathrm{Al}_{11}$ | $2 \cdot 882$ | $0 \cdot 020$ | $\mathrm{Al}_{17}$ | $\begin{array}{ll} 1 \mathrm{Cr}_{1}, & 1 \mathrm{Al}_{3} \\ 1 \mathrm{Al}_{8}, & 1 \mathrm{Al}_{10} \\ 1 \mathrm{Al}_{10}, & 1 \mathrm{Al}_{11} \\ 1 \mathrm{Al}_{12}, & 1 \mathrm{Al}_{14} \\ 1 \mathrm{Al}_{16}, & 1 \mathrm{Al}_{16} \end{array}$ | $1 \mathrm{Al}_{17}$ | $2 \cdot 810$ | $0 \cdot 010$ |
|  |  | $2 \mathrm{Al}_{13}$ | $2 \cdot 834$ | $0 \cdot 020$ |  |  |  |  |  |
|  |  | $2 \mathrm{Al}_{14}$ | $2 \cdot 881$ | $0 \cdot 020$ |  |  |  |  |  |
|  |  | $2 \mathrm{Al}_{15}$ $2 \mathrm{Al}_{15}$ | $2 \cdot 763$ $2 \cdot 905$ | $0 \cdot 020$ 0.020 |  |  |  |  |  |

$\mathrm{Cr}-\mathrm{Al}$ distances are similar to values which occur in $\theta(\mathrm{Cr}-\mathrm{Al})$. The shorter $(2 \cdot 46 \AA)$ occurs in $\mathrm{Cr}_{4} \mathrm{Si}_{4} \mathrm{Al}_{13}$ where each chromium atom has three of these bonds all on the same side, making angles of $63^{\circ}$ to each other. In $\theta(\mathrm{Cr}-\mathrm{Al})$ the two shortest bonds $(2 \cdot 468 \AA$, $2 \cdot 481 \AA$ ) occur from the $\mathrm{Cr}_{2}$ atom, which has three of its four short bonds on the same side, making angles of $70^{\circ}, 70^{\circ}$ and $73^{\circ}$ to each other.

## 10. Summary

The structures of $\theta(\mathrm{Cr}-\mathrm{Al}), \mathrm{Cr}_{4} \mathrm{Si}_{4} \mathrm{Al}_{13}$ and $\mathrm{Mg}_{3} \mathrm{Cr}_{2} \mathrm{Al}_{18}$ all contain short transition metal to aluminium distances similar to those observed in certain other alloy phases and discussed by Taylor (1954) and Brown (1957). The chromium atoms are co-ordinated by nearly regular icosahedra which may interpenetrate, or share corners or faces with other icosahedra. The

Table 3. $\mathrm{Cr}-\mathrm{Al}$ distances and inter-bond angles in $\mathrm{Cr}_{4} \mathrm{Si}_{4} \mathrm{Al}_{13}, \mathrm{Mg}_{3} \mathrm{Cr}_{2} \mathrm{Al}_{18}$ and $\theta(\mathrm{Cr}-\mathrm{Al})$

| Phase | Al atom | Neighbours | Distances ( $\AA$ ) | Angle ( ${ }^{\circ}$ ) |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Cr}_{4} \mathrm{Si}_{4} \mathrm{Al}_{13}$ | $\mathrm{Al}_{1}$ | 2 Cr | $2 \cdot 46$ (2) | 166.2 |
|  | $\mathrm{Al}_{2}$ | 2 Cr | 2.77 (2) | 60.5 |
| $\mathrm{Mg}_{3} \mathrm{Cr}_{2} \mathrm{Al}_{18}$ | $\mathrm{Al}_{f}$ | 2 Cr | 2.58 (2) | $173 \cdot 2$ |
|  | $\mathrm{Al}_{g}$ | 1 Cr | 2.81 |  |
| $\theta(\mathrm{Cr}-\mathrm{Al})$ | $\mathrm{Al}_{0}$ | 4 Cr | $2 \cdot 58$ (4) | 180 (2) |
|  | $\mathrm{Al}_{7}$ | 2 Cr | $2 \cdot 47$ (2) | 167.9 |
|  | $\mathrm{Al}_{8}$ | 2 Cr | $2 \cdot 48,2 \cdot 57$ | 175.0 |
|  | $\mathrm{Al}_{11}$ | 2 Cr | 2.53, $2 \cdot 59$ | 162.7 |
|  | $\mathrm{Al}_{13}$ | 2 Cr | $2 \cdot 48,2 \cdot 57$ | $169 \cdot 4$ |
|  | $\mathrm{Al}_{3}$ | 2 Cr | 2.68 (2) | 59.6 |
|  | $\mathrm{Al}_{6}$ | 2 Cr | $2 \cdot 78$ (2) | 57.2 |
|  | $\mathrm{Al}_{14}$ | 2 Cr | $2 \cdot 80,2 \cdot 91$ | $102 \cdot 4$ |
|  | $\mathrm{Al}_{15}$ | 2 Cr | 2.77, $2 \cdot 79$ | $104 \cdot 9$ |
|  | $\mathrm{Al}_{5}$ | 3 Cr | $2 \cdot 62$ (2), 2.53 | $\leqslant 180$ |
|  | $\mathrm{Al}_{4}$ | 3 Cr | 2.74 (2), 2.86 | $\ll 180$ |
|  | others | 1 Cr | >2.59 | - |

Cr -Al distances in the three phases fall into two groups, one of which is significantly shorter than the sum of the twelve-co-ordinated radii of the atoms. These short distances indicate a stronger $\mathrm{Cr}-\mathrm{Al}$ interaction which is associated with the sharing of corners between the icosahedra when the $\mathrm{Cr}-\mathrm{Al}-\mathrm{Cr}$ bond angle
is near $180^{\circ}$. This suggests a directional character not usually associated with a purely metallic bond.

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## References

Brown, P. J. (1957). Dissertation for the Ph.D. degree, University of Cambridge.
Brown, P. J. (1959). Acta Cryst. 12, 995.
Forsyth, J. B. \& Wells, M. (1959). Acta Cryst. 12, 412. Hofmann, W. \& Wiehr, H. (1941). Z. Metallk. 33, 369. Lipson, H. \& Cochran, W. (1953). The Determination of Crystal Structures, p. 308. London: Bell.
Raynor, G. V. \& Littie, K. (1945). J. Inst. Met. 71, 493. Robinson, K. (1953). Acta Cryst. 6, 854.
Samson, S. (1958). Acta Cryst. 11, 851.
Taylor, W. H. (1954). Acta Met. 2, 684.
Weisz, O., Cochran, W. \& Cole, W. F. (1948). Acta Cryst. 1, 83.

# The Crystal Structure of Sodium Triphosphate, $\mathrm{Na}_{5} \mathrm{P}_{\mathbf{3}} \mathrm{O}_{\mathbf{1}}$, Phase I 

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The crystal structure of the high-temperature form of sodium triphosphate, $\mathrm{Na}_{5} \mathrm{P}_{3} \mathrm{O}_{10}$, Phase I , has been determined by Fourier methods. The unit cell is monoclinic with

$$
a=9 \cdot 61, b=5 \cdot 34, c=19 \cdot 73 \AA ; \beta=112^{\circ} .
$$

The space group is $C 2 / c$ and the unit cell contains 4 units of $\mathrm{Na}_{5} \mathrm{P}_{3} \mathrm{O}_{10}$. The triphosphate ions have twofold axial symmetry with the central phosphorus atoms lying on twofold axes of the unit cell. Bond lengths are $\mathrm{P}-\mathrm{O}$ (chain) $=1.62 \pm 0.03$ (inner), $1.66 \pm 0.03$ (outer), $\mathrm{P}-\mathrm{O}$ (mean, terminal) $1.48 \pm$ $0.03 \AA$. The structure is generally more distorted than that found previously for the low-temperature form, Phase II. Some of the sodium ions are co-ordinated in distorted octahedral arrangements, while the remainder are involved in 4 -fold co-ordination of an unusual type. A comparison of the structures of Phase I and Phase II suggests an explanation of some of their differences in properties.

## Introduction

The crystalline forms of sodium triphosphate $\mathrm{Na}_{5} \mathrm{P}_{3} \mathrm{O}_{10}$ and its hexahydrate are of theoretical interest and of importance in the production of modern detergent powders. A knowledge of the crystal structures of these salts is relevant to the explanation of many of the phenomena connected with their preparation, interconversion and hydration, which are at present
only partially understood (Raistrick, 1948; ; Van Wazer, 1958). In the present paper the structure of the high temperature anhydrous form, Phase I, is worked out and compared with that of the low temperature form, Phase II, which has been given previously (Davies \& Corbridge, 1958).

Small crystals were obtained from a melt of composition $45 \%$ sodium pyrophosphate $\left(\mathrm{Na}_{4} \mathrm{P}_{2} \mathrm{O}_{7}\right)$ and

